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ANALYTIC APPROACHES TO UNSTABLE RESONATORS.(U)
JUN 80 J NAGEL, D ROGOVIN F49620-79-C-0027

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ANALYTIC APPROACHES TO UNSTABLE RESONATORS:

FINAL REPORT

BY

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Abstract:

The marginally stable regime has been distinguished from the unstable regime both qualitatively and quantitatively. New asymptotic techniques for solving the Fresnel-Kirchoff equation for marginally stable resonators were obtained. The method is analytical in nature and yields excellent predictions for the modes and losses of such cavities. Numerical results for both marginally stable and marginally unstable cavities are presented. In addition, conditions for the suitability of using empty resonator modes to describe gain filled cavities have been derived. This deviation is based on manipulations of the differential equation describing gain filled cavities in Hankel transform space. The result is an ordinary, linear Fresnel-Kirchoff integral equation with gain renormalized Fresnel number and eigenvalue.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: AFOSR-TR-80-0623	2. GOVT ACCESSION NO.: AD-A088378	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): ANALYTIC APPROACHES TO UNSTABLE RESONATORS.		5. TYPE OF REPORT & PERIOD COVERED: Final rept.,
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): J. Nagel, D. Rogovin		8. CONTRACT OR GRANT NUMBER(s): F49620-79-C-0027
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Science Applications, Inc., 1200 Prospect Street, La Jolla, CA 92038		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, 2301/A1
11. CONTROLLING OFFICE NAME AND ADDRESS: AFOSR/NP, Bolling AFB, Wash DC 20332		12. REPORT DATE: Jun 80
		13. NUMBER OF PAGES: 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 16		15. SECURITY CLASS. (of this report): unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release;
distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The marginally stable regime has been distinguished from the unstable regime both qualitatively and quantitatively. New asympototic techniques for solving the Fresnel-Kirchoff equation for marginally stable resonators were obtained. The method is analytical in nature and yields excellent predictions for the modes and losses of such cavities. Numerical results for both marginally stable and marginally unstable cavities are presented. In addition, conditions for the suitability of using empty resonator modes to describe gain filled

cavities have been derived. This deviation is based on manipulations of the differential equation describing gain filled cavities in Hankel transform space. The result is an ordinary, linear Fresnel-Kirchoff integral equation with gain renormalized Fresnel number and eigenvalue.

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STATEMENT OF WORK

A. Develop and apply analytic techniques for determining the eigenmodes and eigenvalues of cylindrical unstable optical resonators.

B. For systems which display azimuthal symmetry, develop and apply two different techniques: (1) an asymptotic differential equation approach and (2) an approach that is valid for situations in which the resonator magnification is large.

C. Develop a technique that is designed to treat cylindrical resonators in which the system's azimuthal symmetry has been destroyed by optical perturbations.

D. Develop techniques that are specifically designed for resonators with large Fresnel numbers.

E. Develop a technique to incorporate gain in unstable resonator calculations.

F. Develop analytic techniques for determining the eigenmodes and eigenvalues of finite cylindrical unstable optical resonators with magnification greater than unity.

G. Investigate analytic techniques for incorporating uniformly distributed and saturated gain in the resonators studied under F above.

FINAL REPORT

Unstable resonators have attracted much attention in the last twenty years due to the usefulness of such devices as components in high energy laser systems. The brief advantages of unstable cavities are their large mode volumes, excellent mode selectivities, and efficient output couplings. In order to quantify these factors, it is necessary to accurately describe the distribution of radiation within the cavity.

To do this, it proves sufficient to solve an integral equation which is non-linear in the presence of a saturable gain medium. The logical starting point in the analysis to solve the linear equation which gives the modes of the empty resonator with no gain. This has been accomplished using numerical methods, leading to predictions for the losses, frequency shifts and radiation distributions for a wide range of resonators.

Unfortunately, the numerical solutions are too costly or impossible for cavities with large Fresnel number N, due to the large size of the matrices needed to model the rapidly fluctuating phases. To remedy this situation, asymptotic methods have been developed which lead to solutions for highly unstable resonators with large N. These solutions are not valid for marginally stable cavities with magnification M close to one.

We developed an analytic variational procedure which yields solutions to the empty resonator integral equation for marginally stable cavities with large Fresnel numbers.

To be of practical value, the empty resonator solutions should describe qualitatively the gain filled cavity. We have considered a re-

sonator with a slowly varying gain medium, and shown that the problem can be reduced to solving a linear integral equation of the same form as the empty resonator equation, but with gain renormalized Fresnel number and eigenvalue. This gain renormalized integral equation may be solved using the same techniques developed for empty resonators to yield predictions for losses and intensity distributions for gain filled media.

Alternatively, one may use the expressions for renormalized Fresnel number and eigenvalue to quickly estimate conditions for which the gain medium significantly changes the cavity modes from their empty resonator values.

Below, we summarize our results for empty resonators gain filled systems.

EMPTY RESONATORS

(A) We have found that unstable resonators may be divided into two classes whose characteristics are quantitatively and qualitatively different. These two classes are the marginally stable, with

$$N_{eq} = N\frac{(M^2 - 1)}{2M} \lesssim 1, \qquad (1)$$

and the unstable regime with $N_{eq} > 1$. The Butts-Avizonis-Horwitz asymptotic methods are valid for $N_{eq} > 1$, or

$$M > 1 + N^{-1} \qquad (2)$$

(B) The two classes may be distinguished physically by consideration of the mirror shadow boundary. For marginally stable resonators, the

shadow region of the edge one mirror substantially overlaps the edge of the second mirror. When this occurs, the core term solution described below provides a good approximation to the actual pattern. For unstable resonators with $N_{eq} > 1$, the shadow region of one mirror edge falls outside the opposite mirror. When this occurs, interference due to edge effects is minimized and crossing of eigenmodes begins. In addition, because the shadow region escapes the cavity after each transit of the cavity, repeated propagations may be described rising geometric optics, which makes valid the approach of Butts, Avizonis and Horwitz.

(C) For $N_{eq} < 1$ with $N >> 1$, extending the limits of the integral equation to infinity leads to little error, since most of the radiation leaving one mirror falls on the opposite mirror. The infinite limit integral equation may be solved exactly and analytically to yield the core term solution. The core term solution and its eigenvalue are continuous rather than discrete, because the limits of integration are not finite.

(D) We have shown that the core term solution obtained for unstable cavities may be analytically continued to the stable regime. When boundary conditions at infinity are invoked, it is found that only a finite number of the core term solutions survive, and these are just the usual gaussian-beam solutions.

(E) The accuracy of the core term as a solution is excellent for large N stable cavities, and decreases with increasing unstability. For $N_{eq} > 1$ these solutions are expected to be of little value.

(F) Using the fact that the core term is a reasonably accurate

solution for marginally stable cavities, we have developed a variational approach which chooses the core term solution ϕ_0 which best satisfies the Fresnel-Kirchoff integral equation. This is accomplished by first defining a quantity which measures the error in the integral equation and then minimizing this quantity by varying the continuous parameter which determines the core term. The entire procedure is analytical, and involves finally a solution of one equation in one variable.

(G) The method may be compared with the waveguide analysis used by numerous authors. In this approach, the field near the edge of the mirror is written in terms of incoming and outgoing transverse waves using the first few terms in the asymptotic expansion of the core term solution. The coefficients of these waves, and hence the parameter which determines ϕ_0, are then fixed by comparing with the solution near the edge of a semi-infinite waveguide obtained by Weinstein. The problem with this method is that ϕ_0 is not a valid solution where the asymptotic expansion may be used.

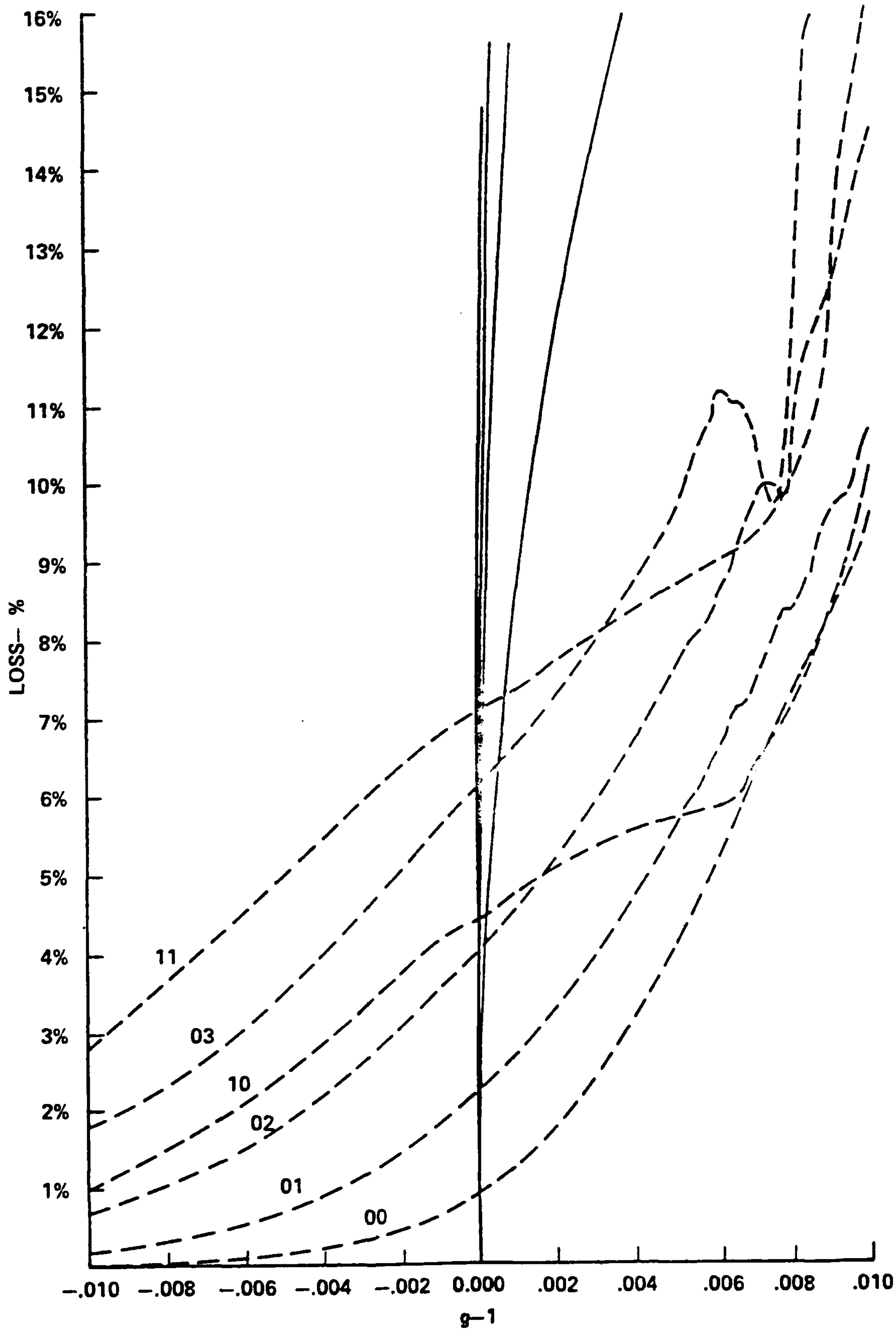
(H) Our method essentially chooses the solution which minimizes the spread of the far field pattern. By contrast, the waveguide method chooses the solution which most closely approximates a waveguide solution near the edge. Since the waveguide solution contains minimal coupling to higher order transverse modes which tend to go around the mirror, this method also leads to a compact far field pattern.

(I) The difficulty in achieving numerical results is proportional to N, rather than log N for asymptotic methods.

(J) Numerical results have been presented for a range of marginally stable and unstable cavities. The method could be used to obtain extremely accurate results for stable cavities. In Figures 1 and 2 we display results for the losses and phase shifts for the six lowest loss modes with $N = 10$. The results are compared with geometric optics predictions.

FIGURE 1: Losses versus mirror curvature for the six lowest loss modes of an N = 10 cavity. The modes are labeled by $n\ell$. The curvature is given in terms of $g - 1 = -L/R$, where R is the mirror curvature which is negative for unstable, convex mirrors. Also shown in solid lines are the geometric optics prediction for the losses. Again, six curves are shown, but the upper four are degenerate.

FIGURE 2: Phase shifts versus mirror curvature for the six lowest loss modes of Fig. 1. Geometric optics-gaussian beam predictions are indicated by solid lines.



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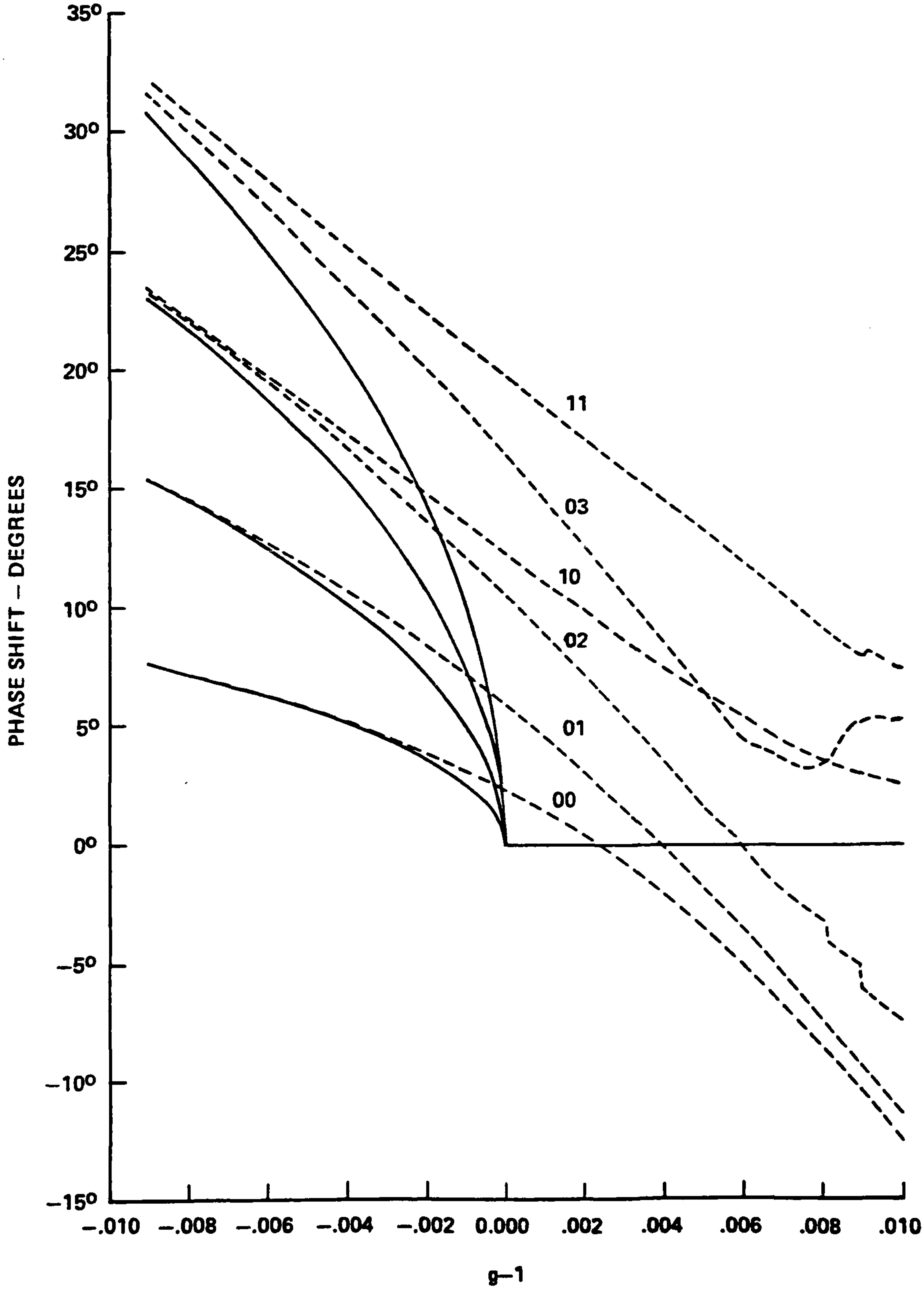

PHASE SHIFT – DEGREES
35°
30°
25°
20°
15°
10°
5°
0°
−5°
−10°
−15°
11
03
10
02
01
00
−.010
−.008
−.006
−.004
−.002
0.000
.002
.004
.006
.008
.010
g−1

GAIN FILLED CAVITIES

(A) The full non-linear integral equation may only be solved numerically. To treat the problem analytically, numerous approximations must be made. A novel aspect of our work has been the fact that we work with the differential equation describing a loaded, cylinderical resonator in the Hankel transform space of the radial coordinate.

(B) All the approximations are made in the evaluation of the Hankel transformed susceptibility. We assume the susceptibilities are slowly varying, then make the Fresnel approximation in Hankel transform space. Finally, the susceptibilities depend on the fields, and these fields are replaced by the empty resonator results.

(C) The result is a Fresnel-Kirchoff integral equation, but with gain renormalized Fresnel number and eigenvalue N_G and λ_G. Although both these quantities are now complex, there is no difficulty in solving the linear integral equation using the usual methods. We have not made the thin gain sheet approximation.

(D) The values of N_G and λ_G are:

$$N_G = N\left[1 + \tfrac{1}{2} igL \frac{b^2}{r_o^2} \left(\frac{2\pi N}{48} - \tfrac{1}{4} i - \frac{1}{6\pi N}\right)\right]^{-1} \qquad (3)$$

$$\lambda_G = \lambda \exp\left[\tfrac{1}{2} gL\left(1 + \tfrac{1}{2}\frac{b^2}{r_o^2}\left(\tfrac{1}{2} - \frac{i}{2\pi N}\right)\right)\right] , \qquad (4)$$

where: L = resonator length

$g = g(o)$ = on axis small signal linear gain

b = resonator radius

r_o = scale of gain variations, $g''(0) \sim r_0^{-2} g(0)$

These relations are not valid for extremely large N, $N > 100$ because of the approximations made.

(E) For homogenous gain, $r_o \rightarrow$ infinity and

$$N_G = N \tag{5}$$

$$\lambda_G = \lambda e^{\frac{1}{2} gL} \tag{6}$$

In this case the intensity pattern is identical with the empty resonator result, and the eigenvalue is modified only by the linear gain.

(F) Typically, $r_o \sim b$ and $gL \sim 0.1$. To significantly change the radiation pattern, one requires ~50% changes in N. Consequently it is easy to see that the cavity modes are not significantly changed by slowly varying gain for $N \ll 100$. To obtain significant differences for $N = 10$, one requires $gL \sim 1$, corresponding to 100% linear gain.

DOCUMENTATION

The research supported by this contract was reported in the following publications:

(1) J. Nagel and D. Rogoivn, "Analytic Approaches to Unstable Resonators", Proceedings Laser '78. (1979).

(2) J. Nagel, D. Rogovin, P. Avizonis and R. Butts, "Asymptotic Approaches to Marginally Stable Resonators", Optics Letters 4, p. 300. (1979).

(3) J. Nagel, D. Rogovin, and P. Avizonis, "The Effect of Gain on the Electrodynamics of Marginally Stable Resonators", Optics Letters 5, p. 90 (1980).

(4) J. Nagel and D. Rogovin, "Iterative Methods for Marginally Stable Resonators", Proceedings Laser '79. (1980).

(5) J. Nagel and D. Rogovin, "Analytic Approaches to Unstable Resonators: Annual Technical Report." (1979).

(6) J. Nagel and D. Rogovin, "An Analytical Variational Method for Obtaining the Modes of Marginally Stable Resonators", submitted to JOSA. (1980).

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Ph.D., University of Pennsylvania, 1970
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2. "Analytic Approaches to Unstable Optical Resonators"
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3. "Iterative Methods for Marginally Stable Resonators"
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(B) Consultive and Advisory Functions:

AFWL personnel who were closely associated with this project included P. Avizonis, G. Dente, A. Paxton and R. Butts.